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Datta–Das transistor with enhanced spin control

J. Carlos Egues,[a)] Guido Burkard, and Daniel Loss
Department of Physics and Astronomy, University of Basel, Klingelbergstrasse 82, CH-4056 Basel, Switzerland

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We consider a two-channel spin transistor with weak spin-orbit induced interband coupling. We show that the coherent transfer of carriers between the coupled channels gives rise to an additional spin rotation. We calculate the corresponding spin-resolved current in a Datta–Das geometry and show that a weak interband mixing leads to enhanced spin control. [DOI: 10.1063/1.1564867]

The pioneering spin-transistor proposal of Datta and Das[1] best exemplifies the relevance of electrical control of magnetic degrees of freedom as a means of spin modulating charge flow. In this device,[2] a spin-polarized current[3,4] injected from the source is spin modulated on its way to the drain via the Rashba spin-orbit[5] (s-o) interaction, Fig. 1(a). The spin transistor operation relies on gate controlling[6] the strength α of the Rashba interaction which has the form $H_R = i\alpha\sigma_y \partial/\partial x$ in a strictly one-dimensional channel.[5] Upon crossing the Rashba-active region of length L, a spin-up incoming electron emerges in the spin-rotated state

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{bmatrix} \cos(\theta_R/2) \\ -\sin(\theta_R/2) \end{bmatrix}, \tag{1}$$

where $\theta_R = 2m^*\alpha L/\hbar^2 \equiv 2k_R L$ is the rotation angle and m^* is the electron effective mass.[1] The corresponding spin-resolved conductance is found to be $G_{\uparrow,\downarrow} = e^2(1 \pm \cos\theta_R)/h$.

Here we extend the above picture by considering a geometry with two weakly coupled Rashba bands in the quasione-dimensional channel, Fig. 1(b). We treat the degenerate k states near the band crossings perturbatively in analogy to the nearly free electron model.[7] This approach allows for a simple analytical description of the problem. We calculate the spin-resolved current by extending the usual procedure of Datta and Das[1] to account for weakly coupled bands. Our main finding is an additional spin rotation for injected electrons with energies near the band crossing (see shaded region around ε_F in Fig. 2). As we derive later on, an incoming spin up electron in channel a emerges from the Rashba region in the rotated state

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \rightarrow \frac{1}{2} \begin{bmatrix} \cos(\theta_d/2)e^{-ik_R L} + e^{ik_R L} \\ -i\cos(\theta_d/2)e^{-ik_R L} + ie^{ik_R L} \\ -i\sin(\theta_d/2)e^{-ik_R L} \\ \sin(\theta_d/2)e^{-ik_R L} \end{bmatrix}, \tag{2}$$

where $\theta_d = \theta_R d/k_c$ is the additional spin rotation angle, d is the interband matrix element, and k_c is the wave vector at the band crossing, Fig. 2. From Eq. (2) we can find the spin-resolved conductance

$$G_{\uparrow,\downarrow} = \frac{e^2}{h}\begin{bmatrix} 1 + \cos(\theta_d/2)\cos\theta_R \\ 1 - \cos(\theta_d/2)\cos\theta_R \end{bmatrix}. \tag{3}$$

We now proceed to derive Eqs. (2) and (3).

Model. We consider a quasione-dimensional wire of length L with two bands a and b described by $\varepsilon_{n,\sigma_z}(k) = \hbar^2 k^2/2m^* + \epsilon_n$, $n = a,b$ and eigenfunctions $\varphi_{k,n,\sigma}(x,y) = e^{ikx}\phi_n(y)|\sigma\rangle/\sqrt{L}$, $\sigma = \uparrow,\downarrow$ where the $\phi_n(y)$'s denote the transverse confinement wave functions. In the presence of the Rashba s-o interaction, we can derive a Hamiltonian for the system in the basis of the uncoupled wave functions $[\varphi_{k,n,\sigma_z}(x,y)]$. This reads

$$H_R = \begin{bmatrix} \varepsilon_+^a(k) & 0 & 0 & -\alpha d \\ 0 & \varepsilon_-^a(k) & \alpha d & 0 \\ 0 & \alpha d & \varepsilon_+^b(k) & 0 \\ -\alpha d & 0 & 0 & \varepsilon_-^b(k) \end{bmatrix}, \tag{4}$$

where $d \equiv \langle \phi_a(y) | \partial/\partial y | \phi_b(y) \rangle$, $\varepsilon_s^n(k) = \hbar^2(k - sk_R)^2/2m^* + \epsilon_n - \epsilon_R$, $\epsilon_R \equiv \hbar^2 k_R^2/2m^*$, $(s = \pm, n = a,b)$ and we have considered $|\sigma\rangle$ to be the eigenbasis of σ_y. For $d = 0$ the Hamiltonian in Eq. (4) is diagonal and yields uncoupled Rashba dispersions $\varepsilon_s^n(k)$ (thin lines in Fig. 2); the corresponding wave functions are $\varphi_{k,n,s}(x,y)$ (here $|\sigma\rangle \rightarrow |s = \pm\rangle = [|\uparrow\rangle \pm i|\downarrow\rangle]/\sqrt{2}$). Note that for $d = 0$ the bands cross for some values of k. For instance, for $k > 0$ a crossing occurs at $k_c = (\epsilon_b - \epsilon_a)/2\alpha$. For nonzero interband coupling $d \neq 0$,[8] we can diagonalize H_R exactly (see Mireles and Kirczenow in Ref. 8) to find the dispersions (thick lines in Fig. 2).

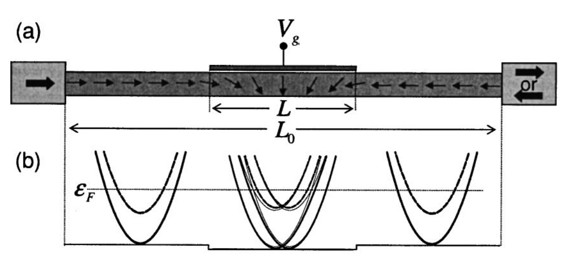

FIG. 1. Spin transistor geometry with a two-band channel. (a) The length L of the Rashba region is smaller than the total length L_0 of the wire. (b) Sketch of energy dispersions in the s-o active region with and without interband coupling (Rashba bands) and away from it (parabolic bands). Note the small band offsets between adjacent regions in the wire.

[a)]Also at: Department of Physics and Informatics, University of São Paulo at São Carlos, 13560-970 São Carlos/SP, Brazil; electronic mail: egues@if.sc.usp.br

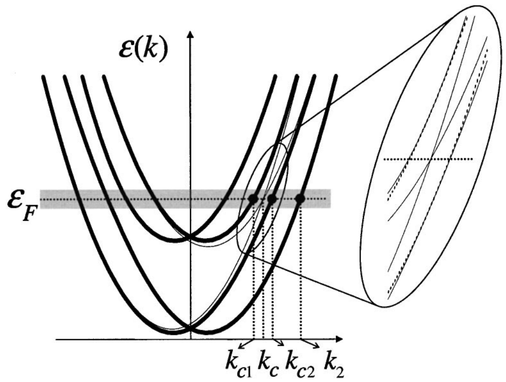

FIG. 2. Band structure in the presence of spin-orbit coupling. In absence of interband mixing the Rashba dispersions are uncoupled (thin solid lines) and cross at, e.g., k_c. For nonzero interband coupling the bands anti cross (thick solid lines). The inset shows a blowup of the dispersion region near the crossing: the approximate solution [dotted lines, perturbative approach, Eq. (6)] describes well the energy dispersions near k_c.

Bands near k_c. Since we are interested in transport with injection energies near the crossing, we follow here a simpler perturbative approach[7] to determine the energy dispersions and wave functions near k_c. Near the crossing we can solve the reduced Hamiltonian

$$\tilde{H}_R=\begin{bmatrix} \varepsilon_-^a(k) & \alpha d \\ \alpha d & \varepsilon_+^b(k) \end{bmatrix}, \tag{5}$$

which to lowest order yields

$$\varepsilon_\pm^{\text{approx}}(k)=\frac{\hbar^2k^2}{2m^*}+\frac{1}{2}\epsilon_b+\frac{1}{2}\epsilon_a\pm\alpha d. \tag{6}$$

As shown in the inset of Fig. 2, Eq. (6) describes very well the anticrossing of the bands near k_c. The corresponding zero-order eigenstates are

$$|\psi_\pm\rangle=\frac{1}{\sqrt{2}}[|-\rangle_a\pm|+\rangle_b]=\frac{1}{\sqrt{2}}\left[\begin{pmatrix}1\\-i\end{pmatrix}_a\pm\begin{pmatrix}1\\i\end{pmatrix}_b\right], \tag{7}$$

where the subindices indicate the respective channel. The analytical form in Eq. (6) allows us to determine the wave vectors k_{c1} and k_{c2} in Fig. 2 straightforwardly: we assume $k_{c1}=k_c-\Delta/2$ and $k_{c2}=k_c+\Delta/2$ and solve $\varepsilon_+^{\text{approx}}(k_{c1})=\varepsilon_-^{\text{approx}}(k_{c2})$ (assumed $\sim\varepsilon_F$) to find

$$\Delta=\frac{2m^*\alpha d}{\hbar^2k_c}=2\frac{k_R}{k_c}d. \tag{8}$$

Note that to the lowest order used here the horizontal splitting Δ is constant and symmetric about k_c.

Boundary conditions. We now consider a spin-up electron entering the Rashba-active region of length L in the wire. Following the usual approach, we expand this incoming state in terms of the coupled Rashba states in the wire. We consider only the states k_{c1}, k_{c2}, and k_2 in the expansion

$$|\Psi\rangle=\tfrac{1}{2}|\psi_+\rangle e^{ik_{c1}x}+\tfrac{1}{2}|\psi_-\rangle e^{ik_{c2}x}+\tfrac{1}{\sqrt{2}}|+\rangle_a e^{ik_2x}. \tag{9}$$

The above ansatz satisfies the boundary conditions for both the wave function and (to leading order) its derivative at $x=0$. More explicitly, the velocity operator condition[9] at $x=0$ for an electron with $k=k_F$ yields

$$\begin{pmatrix}k_F\\0\\0\\0\end{pmatrix}=\frac{1}{2}\begin{bmatrix}k_c+k_2\\-i(k_c-k_2-2k_R)\\-\Delta/2\\-i\Delta/2\end{bmatrix}=\frac{1}{2}\begin{pmatrix}k_c+k_2\\0\\-\Delta/2\\-i\Delta/2\end{pmatrix}, \tag{10}$$

where we used $k_2-k_c=2k_R$ (still valid to leading order). The "four-vector" notation in Eq. (10) concisely specifies the spin states in channels a (upper half) and b (lower half). Note that Eq. (10) is satisfied provided that $\Delta\ll 4k_F$. This inequality is satisfied in our system for realistic parameters.

Underlying the ansatz in Eq. (9) is the assumption of unity transmission through the Rashba region. Here we have in mind the particular spin-transistor geometry sketched in Fig. 1(a): a gate-controlled Rashba-active region of extension L smaller than the total length L_0 of the wire. In this configuration, there are only small band offsets (which we neglect) of the order of $\epsilon_R\ll\varepsilon_F$ at the entrance $(x=0)$ and exit $(x=L)$ of the Rashba region. Hence, transmission is indeed very close to unity, see Ref. 10. The boundary conditions at $x=L$ are also satisfied.

Generalized spin-rotated state. From Eq. (9) we find that a spin-up electron entering the Rashba region at $x=0$ emerges from it at $x=L$ in the spin-rotated state

$$\begin{aligned}\Psi_{\uparrow,L}&=\frac{1}{4}\left[\begin{pmatrix}e^{-iL\Delta/2}\\-ie^{-iL\Delta/2}\\e^{-iL\Delta/2}\\ie^{-iL\Delta/2}\end{pmatrix}+\begin{pmatrix}e^{iL\Delta/2}\\-ie^{iL\Delta/2}\\-e^{iL\Delta/2}\\-ie^{iL\Delta/2}\end{pmatrix}\right]e^{ik_cL}\\&\quad+\frac{1}{2}\begin{pmatrix}1\\i\\0\\0\end{pmatrix}e^{ik_2L}\\&=\frac{1}{2}e^{i(k_c+k_R)L}\begin{bmatrix}\cos(\theta_d/2)e^{-ik_RL}+e^{ik_RL}\\-i\cos(\theta_d/2)e^{-ik_RL}+ie^{ik_RL}\\-i\sin(\theta_d/2)e^{-ik_RL}\\\sin(\theta_d/2)e^{-ik_RL}\end{bmatrix},\end{aligned} \tag{11}$$

which is essentially Eq. (2). Observe that in absence of interband coupling (i.e., $\theta_d=0$) Eq. (11) reduces to the Datta–Das state in Eq. (1). An expression similar to Eq. (11) holds for the case of an incoming spin-down electron.

Spin-resolved current. For $x\geqslant L$ we have

$$\begin{aligned}\Psi_\uparrow(x\geqslant L,y)&=\frac{1}{2}\begin{bmatrix}e^{-i\theta_R/2}\cos(\theta_d/2)+e^{i\theta_R/2}\\-ie^{-i\theta_R/2}\cos(\theta_d/2)+ie^{i\theta_R/2}\end{bmatrix}e^{i(k_c+k_R)x}\phi_a(y)\\&\quad+\frac{1}{2}\begin{bmatrix}-ie^{i\theta_R/2}\sin(\theta_d/2)\\e^{i\theta_R/2}\sin(\theta_d/2)\end{bmatrix}e^{i(k_c-k_R)x}\phi_b(y),\end{aligned} \tag{12}$$

which describes planes waves in the uncoupled channels a and b arising for an incoming spin-up electron in channel a. The total current follows straightforwardly (Landauer–Büttiker) from Eq. (12):

$$I_{\uparrow,\downarrow}=\frac{e}{h}eV[1\pm\cos(\theta_d/2)\cos\theta_R]. \tag{13}$$

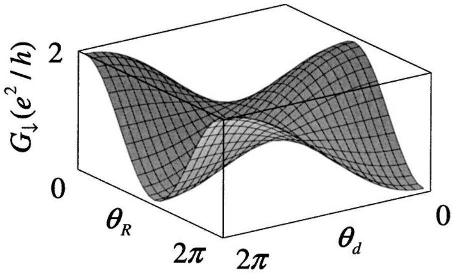

FIG. 3. Angular dependence of the spin-down conductance. The additional modulation θ_d due to s-o interband mixing and θ_R can be varied independently.

where $eV \ll \varepsilon_F$ the applied bias between the source and drain. The spin-dependent conductance in Eq. (3) follows immediately from Eq. (13). Equation (13) clearly shows the additional modulation θ_d of the spin-resolved current due to s-o induced interband coupling. Figure 3 illustrates the angular dependence of $G_\downarrow$ as a function of θ_R and θ_d. The s-o mixing angle θ_d enhances the possibilities for spin control in the Datta–Das transistor.

Realistic parameters. For concreteness, let us consider infinite transverse confinement (width w). In this case, $\epsilon_b - \epsilon_a = 3\hbar^2\pi^2/2mw^2$ and the interband coupling constant $d = 8/3w$. We choose $\epsilon_b - \epsilon_a = 16\epsilon_R$, which implies (i) $\alpha = (\sqrt{3}\pi/4)\hbar^2/mw = 3.45\times 10^{-11}$ eV m (and $\epsilon_R \sim 0.39$ meV) for $m = 0.05m_0$ and $w = 60$ nm, (ii) $\varepsilon(k_c) = 24\epsilon_R$ [ε_F should be tuned to $\sim\varepsilon(k_c)$], and (iii) $k_c = 8\epsilon_R/\alpha$. Assuming $L = 69$ nm [Rashba region length, Fig. 1(a)], we find $\theta_R = \pi$ and $\theta_d = \theta_R d/k_c = \pi/2$, since $d/k_c \sim 0.5$. This is a conservative estimate. In principle, θ_d can be varied independently of θ_R via lateral gates which alter w. Note also that $\Delta/4k_F \sim 0.05$ [validity of Eq. (10)] for the previous parameters. Finally, we note that the most relevant spin-flip mechanism (Dyakonov–Perel) should be suppressed in quasione-dimensional systems such as ours.[11] In addition, thermal effects are irrelevant in the experimentally feasible linear regime[12] we consider here.

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