Quantum Information Processing Using Quantum Dot Spins and Cavity QED

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The electronic spin degrees of freedom in semiconductors typically have decoherence times that are several orders of magnitude longer than other relevant time scales. A solid-state quantum computer based on localized electron spins as qubits is therefore of potential interest. Here, a scheme that realizes controlled interactions between two distant quantum dot spins is proposed. The effective long-range interaction is mediated by the vacuum field of a high finesse microcavity. By using conduction-band-hole Raman transitions induced by classical laser fields and the cavity-mode, parallel controlled-not operations, and arbitrary single qubit rotations can be realized.

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Within the last few years, quantum computation (QC) has developed into a truly interdisciplinary field involving the contributions of physicists, engineers, and computer scientists [1]. The seminal discoveries of Shor and others, both in developing quantum algorithms for important problems like prime factorization [2], and in developing protocols for quantum error correction (QEC) [3] and fault-tolerant quantum computation [4], have indicated the desirability and the ultimate feasibility of the experimental realization of QC in various quantum systems.

The elementary unit in most QC schemes is a two-state system referred to as a quantum bit (qubit). Since QEC can work only if the decoherence rate is small, it is crucial to identify schemes where the qubits are well isolated from their environment. Ingenious schemes based on Raman-coupled low-energy states of trapped ions [5] and nuclear spins in chemical solutions [6] satisfy this criterion, in addition to providing methods of fast quantum manipulation of qubits that do not introduce significant decoherence. Even though these schemes are likely to provide the first examples of quantum information processing at a 5–10 qubit level, they do not appear to be scalable to larger systems containing more than 100 qubits.

Here, we propose a new scheme for quantum information processing based on quantum dot (QD) electron spins coupled through a microcavity mode. The motivation for this scheme is threefold: (1) a QC scheme based on semiconductor quantum dot arrays should be scalable to $\geq$100 coupled qubits; (2) recent experiments demonstrated very long spin decoherence times for conduction-band electrons in III-V and II-VI semiconductors [7], making electron spin a likely candidate for a qubit; and (3) cavity-QED techniques can provide long-distance, fast interactions between qubits [8]. The QC scheme detailed below relies on the use of a single cavity mode and laser fields to mediate coherent interactions between distant QD spins. As we will show shortly, the proposed scheme does not require that QDs be identical and can be used to carry out parallel quantum logic operations [9]. Even though the QD and microcavity parameters that we assume have already been achieved in various systems, the actual physical implementation of the proposed scheme would still be experimentally quite demanding.

We note that a QC scheme based on electron spins in QDs has been previously proposed [10]: this scheme is based on local exchange interactions controlled by electrodes. The possibility of coherently manipulating motional degrees of freedom of QD electrons using terahertz cavity QED has also been discussed [11]. The principal feature that distinguishes the present proposal from its predecessors is the use of all-optical Raman transitions to couple two conduction-band spin states: this potentially enables us to combine the ultralong spin coherence times with fast, long-distance, parallel optical switching.

The proposed scheme is detailed in Fig. 1: the doped QDs are embedded in a microdisk structure with diameter $d \simeq 2\mu$ and thickness $d \simeq 0.1\mu$. Motivation for choosing this structure over other alternatives comes from recent experiments demonstrating that InAs self-assembled QDs can be embedded in microdisk structures with a cavity quality factor $Q \simeq 12\,000$ [12]. We assume that the QDs are designed such that the quantum confinement along the z direction is the strongest: this is the case both in QDs defined by electrical gates and in self-assembled QDs. The in-plane confinement is also assumed to be large enough to guarantee that the electron will always be in the ground-state orbital. Because of the strong z-axis confinement, the lowest energy eigenstates of such a III-V or II-VI semiconductor QD consist of $|m_z = \pm 1/2\rangle$ conduction-band states and $|m_z = \pm 3/2\rangle$ valence-band states. The QDs are doped such that each QD has a full valence band and a single conduction-band electron: we assume that a uniform magnetic field along the x direction (B_x) is applied, so the QD qubit is defined by the conduction-band states $|m_x = -1/2\rangle = |\downarrow\rangle$ and $|m_x = 1/2\rangle = |\uparrow\rangle$ (Fig. 2). The corresponding energies are $\hbar\omega_\downarrow$ and $\hbar\omega_\uparrow$, respectively.

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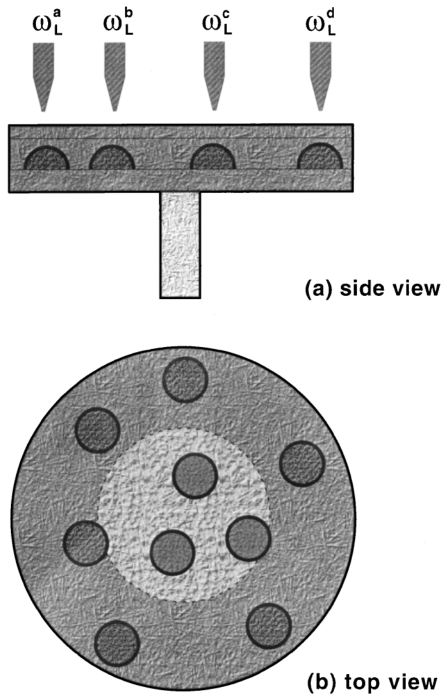

FIG. 1. The quantum information processing scheme that is based on quantum dots embedded inside a microdisk structure. Each quantum dot is addressed selectively by a laser field from a fiber tip. The laser frequencies are chosen to select out the pair of quantum dots that will participate in gate operation. All dots strongly couple to a single cavity mode.

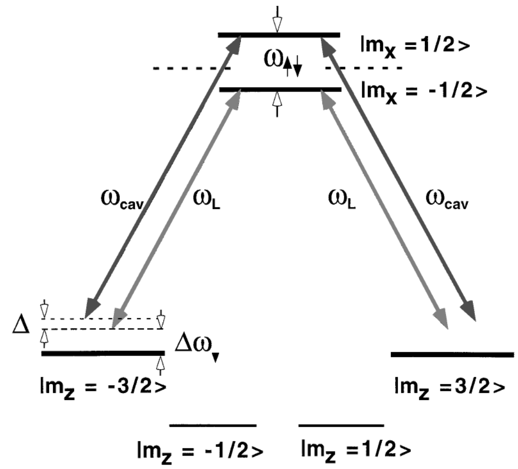

FIG. 2. The relevant energy levels of a III-V (or II-VI) semiconductor quantum dot. It is assumed that confinement along the z direction is strongest.

One of the key elements of the proposed scheme is the Raman coupling of the two spin eigenstates via strong laser fields and a single microcavity mode. Since the QD array dimensions are assumed to be on the order of several microns, we assume that the laser fields are coupled selectively to given QDs using near-field techniques; e.g., via tapered fiber tips. The one-bit operations proceed by applying two laser fields $E_{L,x}(t)$ and $E_{L,y}(t)$ with Rabi frequencies $\Omega_{L,x}$ and $\Omega_{L,y}$, and frequencies $\omega_{L,x}$ and $\omega_{L,y}$ (polarized along the x and y directions, respectively) that exactly satisfy the Raman-resonance condition between $|\downarrow\rangle$ and $|\uparrow\rangle$. The laser fields are turned on for a short time duration that satisfies a π/r-pulse condition, where r is any real number. The process can be best understood as a Raman π/r-pulse for the *hole* in the conduction-band state. The laser field polarizations should have nonparallel components in order to create a nonzero Raman coupling (if there is no heavy-hole light-hole mixing). These arbitrary single-bit rotations can naturally be carried out in parallel. In addition, the QDs that are not doped by a single electron never couple to the Raman fields and can safely be ignored.

The fundamental step in the implementation of two-qubit operations is the selective coupling between the *control* and *target* qubits that is mediated by the microcavity field. To this end, we assume that the cavity mode with energy $\hbar\omega_{\mathrm{cav}}$ (assumed to be x polarized) and a laser field (assumed to be y polarized) establishes the Raman transition between the two conduction-band states, in close analogy with the atomic cavity-QED schemes [8]. After eliminating the valence-band states [13], we obtain the effective Hamiltonian ($\hbar = 1$)

$$\hat{H}_{\mathrm{eff}} = \omega_{\mathrm{cav}}\hat{a}^\dagger_{\mathrm{cav}}\hat{a}_{\mathrm{cav}} + \sum_i \left[\omega^i_{\uparrow\downarrow}\hat{\sigma}^i_{\uparrow\uparrow} - \frac{g^2_{\mathrm{cav}}}{\Delta\omega^i_\uparrow}\hat{\sigma}^i_{\downarrow\downarrow}\hat{a}^\dagger_{\mathrm{cav}}\hat{a}_{\mathrm{cav}} - \frac{(\Omega^i_{L,y})^2}{\Delta\omega^i_\downarrow}\hat{\sigma}^i_{\uparrow\uparrow} + i g^i_{\mathrm{eff}}[\hat{a}^\dagger_{\mathrm{cav}}\hat{\sigma}^i_{\downarrow\uparrow}e^{-i\omega^i_{L,y}t} - \mathrm{H.c.}] \right], \quad (1)$$

where

$$g^i_{\mathrm{eff}}(t) = \frac{g_{\mathrm{cav}}\Omega^i_{L,y}(t)}{2}\left(\frac{1}{\Delta\omega^i_\uparrow} + \frac{1}{\Delta\omega^i_\downarrow}\right) \quad (2)$$

is the effective two-photon coupling coefficient for the spins of the ith QD. $\hat{a}_{\mathrm{cav}}$ denotes the cavity-mode annihilation operator and $\hat{\sigma}^i_{\uparrow\downarrow} = |\uparrow\rangle\langle\downarrow|$ is the spin projection operator for the ith QD. The exact two-photon resonance condition would be $\Delta\omega^i_\uparrow = \omega^i_\uparrow - \omega^i_v - \omega_{\mathrm{cav}} = \Delta\omega^i_\downarrow = \omega^i_\downarrow - \omega^i_v - \omega^i_L$. $\hbar\omega^i_v$ denotes the energy of the relevant valence-band states [14], and $\omega^i_{\uparrow\downarrow} = \omega^i_\uparrow - \omega^i_\downarrow$. The derivation of $\hat{H}_{\mathrm{eff}}$ assumes $\Delta\omega^i_{\uparrow,\downarrow} \gg g_{\mathrm{cav}}, \omega^i_{\uparrow\downarrow} \gg k_B T$, and $\omega^{i,j}_{\uparrow\downarrow} \gg g^i_{\mathrm{eff}} > \Gamma_{\mathrm{cav}}$, where Γ_{cav} denotes the cavity decay rate [not included in Eq. (1)]. The third and fourth terms of Eq. (1) describe the ac-Stark effect caused by the cavity and laser fields, respectively.

The first step in the implementation of a CNOT operation would be to turn on laser fields ω^i_L and ω^j_L to establish a near two-photon resonance condition for both the control (i) and the target (j) qubits,

$$\Delta_i = \omega^i_{\uparrow\downarrow} - \omega_{\mathrm{cav}} + \omega^i_L = \Delta_j \ll \omega^{i,j}_{\uparrow\downarrow}. \quad (3)$$

If we choose two-photon detunings Δ_i large compared to the cavity linewidth and $g^i_{\rm eff}(t)$, we can eliminate the cavity degrees of freedom [13] to obtain an effective two-qubit interaction Hamiltonian in the rotating frame (interaction picture with $H_0 = \sum_i \omega^i_{\uparrow\downarrow}\hat{\sigma}^i_{\uparrow\uparrow}$),

$$\hat{H}^{(2)}_{\rm int} = \sum_{i\neq j} \frac{\tilde{g}_{ij}(t)}{2}\,[\hat{\sigma}^i_{\uparrow\downarrow}\hat{\sigma}^j_{\downarrow\uparrow}e^{i\Delta_{ij}t} + \hat{\sigma}^j_{\uparrow\downarrow}\hat{\sigma}^i_{\downarrow\uparrow}e^{-i\Delta_{ij}t}], \quad (4)$$

where $\tilde{g}_{ij}(t) = g^i_{\rm eff}(t)g^j_{\rm eff}(t)/\Delta_i$ and $\Delta_{ij} = \Delta_i - \Delta_j$. Equation (4) is one of the principal results of this Letter: it shows that Raman coupling via a common cavity mode can establish fully controllable long-range transverse spin-spin interactions between two distant QD electrons, by choosing ω^i_L and ω^j_L such that $\Delta_{ij} = 0$. We can see from $\hat{H}^{(2)}_{\rm int}$ that the size nonuniformity of QDs is in principle completely irrelevant for the proposed scheme: small differences in the g factor between different QDs can be adjusted by the choice of ω^i_L and ω^j_L. The additional single-spin terms of the effective interaction Hamiltonian [not shown in Eq. (4)] induce single qubit phase shifts; however, these terms can be safely discarded since they are smaller than the ac-Stark terms of Eq. (1).

Next, we turn to the implementation of the conditional phase-flip (CPF) operation between spins i and j: to this end, we set $\Delta_{ij} = 0$ and obtain

$$\hat{H}^{(2)}_{{\rm int},ij} = \frac{\tilde{g}_{ij}(t)}{2}\,[\hat{\sigma}^i_y\hat{\sigma}^j_y + \hat{\sigma}^i_z\hat{\sigma}^j_z], \quad (5)$$

where $\hat{\sigma}_y$ and $\hat{\sigma}_z$ are the Pauli operators. In this form, we recognize that the effective interaction Hamiltonian between two QDs is that of transverse spin-spin coupling. Previously, the possibility of carrying out universal QC has been shown for Hamiltonians of the form $H \sim J\mathbf{S}^i \cdot \mathbf{S}^j$ [10]. For the transverse spin-spin coupling of Eq. (5), we find that a nontrivial two-qubit gate, such as the CPF operation, can be carried out by combining $\hat{H}^{(2)}_{{\rm int},ij}$ with one-bit rotations. The unitary evolution operator under the Hamiltonian of Eq. (5) is

$$\hat{U}_{XY}(\phi) = T\exp\left[i\int dt\,\hat{H}^{(2)}_{{\rm int},ij}\right], \quad (6)$$

where $\phi = \int dt\,\tilde{g}_{ij}(t)$. The CPF gate ($\hat{U}_{\rm CPF}$) can be realized by the sequence of operators

$$\hat{U}_{\rm CPF} = e^{i\pi/4}e^{i\pi\mathbf{n}_i\cdot\boldsymbol{\sigma}_i/3}e^{i\pi\mathbf{n}_j\cdot\boldsymbol{\sigma}_j/3}\hat{U}_{XY}(\pi/4) \times e^{i\pi\sigma^i_z/2}\hat{U}_{XY}(\pi/4)e^{-i\pi\sigma^i_y/4}e^{-i\pi\sigma^j_y/4}, \quad (7)$$

where $\boldsymbol{\sigma}$ denotes the vector Pauli operator, $\mathbf{n}_i = (1,1,-1)/\sqrt{3}$, and $\mathbf{n}_j = (-1,1,1)/\sqrt{3}$. Note that all operators in this sequence are understood in the interaction picture defined above in Eq. (4). The controlled-not gate can be realized by combining the CPF operation with single qubit rotations $\hat{U}_{\rm CNOT} = \exp[-i\pi\sigma^j_z/4]\hat{U}_{\rm CPF} \times \exp[i\pi\sigma^j_z/4]$.

Equation (4) also indicates that two-qubit interactions such as the CPF operation can be carried out in parallel. To see how this works, we consider for simplicity four QDs where we set $\Delta_a = \Delta_c$ and $\Delta_b = \Delta_d$ and choose $\Delta_{ab} \gg \tilde{g}_{ij}(t)$ ($i,j = a,b,c,d$; $i \neq j$), by adjusting the corresponding laser frequencies (Fig. 1). For these parameters, QDs a and c (as well as b and d) will couple to each other via the transverse spin-spin interaction of Eq. (5), whereas the coupling between all other pairings of QDs will be energy nonconserving and average out to zero. Generalizing this procedure, we find that a single cavity mode can be used to carry out many parallel two-qubit operations. An analogous method to achieve parallel operations in ion-trap QC was described in Ref. [9].

The time required to complete a two-qubit gate operation in this scheme will be limited by the strength of the electron-hole-cavity coupling. One-bit operations can be completed in a 10 psec time scale, assuming $\Omega_{L,x}, \Omega_{L,y} \simeq 1$, and $\Delta\omega^i_\uparrow \simeq \Delta\omega^i_\downarrow \simeq 5$ meV. If we assume $g_{\rm cav} \simeq \Delta_i \simeq 0.5$, we find $\tilde{g}_{ij} \simeq 0.02$ meV and that CPF operation can be carried out in $\sim$100 psec. For InAs self-assembling QDs, $g_{\rm cav} \simeq 0.1$ meV for a cavity volume $V_{\rm cav} = 4(2\pi c_n/\omega_{\rm cav})^3$, where c_n is the speed of light in the medium. It should be possible to obtain $g_{\rm cav} \simeq 0.5$ meV by utilizing large area QDs [15].

One of the key issues in the feasibility of a quantum information processing scheme is the relative magnitude of the decoherence rates as compared to the gate-operation time. As indicated earlier, our scheme is motivated by the 1 μsec long coherence times of conduction-band electrons observed in doped QW and bulk semiconductors [7]. Recent experiments in undoped QDs, on the other hand, indicate that spin decoherence times are at least as long as 3 nsec even in the presence of valence-band holes [17]; the corresponding times for undoped QW structures are on the order of 50 psec. It should therefore be safe to assume that at least in an ideal system such as GaAs QDs embedded in AlGaAs, the spin decoherence times will be around 1 μsec.

The principal spin decoherence mechanism in QDs is the strong spin-orbit interaction in the valence band. Presence of (valence-band) holes therefore could potentially increase the decoherence rate by several orders of magnitude. By utilizing the Raman scheme, we create only *virtual holes* with a probability of $\simeq 0.01$ for the assumed values of $\Omega_{L,y}, \Delta\omega^i_\uparrow, \Delta\omega^i_\downarrow$. If the hole-spin decoherence time is 1 nsec, this could give rise to an effective decoherence time of 100 nsec; we reiterate however that this gate error appears only in the presence of the laser fields. In the cavity-QED scheme that we propose, the finite cavity lifetime ($\Gamma^{-1}_{\rm cav} \sim 10$ psec) also introduces a decoherence mechanism. However, since the cavity mode is also only virtually excited during the transverse spin-spin interaction (with probability $\sim$0.01 for the assumed parameters), the effective decoherence time will be on the order of 1 nsec. Therefore, the primary technological limitation for the proposed scheme is the relatively fast photon loss rate. This rate can be improved either by new processing techniques for microdisk structures or by coupling QDs to ultrahigh finesse cavities of silica microspheres [16].

Finally, the accurate measurement of the spin state of each qubit is an essential requirement in a QC scheme. In our system, this can be achieved by applying a laser field $E_{L,y}$ to the QD to be measured, in order to realize exact two-photon resonance with the cavity mode. If the QD spin is in state $|\downarrow\rangle$, there is no Raman coupling and no photons will be detected. If, on the other hand, the spin state is $|\uparrow\rangle$, the electron will exchange energy with the cavity mode and eventually a single photon will leak out of the cavity. A single photon detection capability is sufficient for detecting a single spin.

One of the key assumptions of our scheme is the need to address each QD selectively. Using state-of-the-art near-field techniques can give us resolution that is on the order of 1000 Å, which in turn gives an upper limit of ~ 10 as the total number of QDs that can be coupled to a single cavity mode. An alternative method to couple larger number of QDs could be to use the electric field dependence of the electron g factor $g_{\rm elec}$ in semiconductors [18]. An ideal situation for the proposed scheme is to design the system such that each QD will have a dedicated gate electrode that can be used to switch its $g_{\rm elec}$ and hence bring the QD in and out of Raman resonance with the laser and cavity modes [11]. The laser fields can still be pulsed, but need not be localized spatially.

In summary, we have described a new quantum information processing scheme based on QDs strongly coupled to a microcavity mode. The primary achievement of this scheme is the realization of parallel, long-range transverse spin-spin interactions between conduction-band electrons, mediated by the cavity mode. Such interactions can be used to carry out quantum computation. Currently, the primary technological limitation of the proposed scheme is the short photon lifetime in state-of-the-art microcavities. In addition, the requirement for addressing each QD individually strongly limits the scalability. Further improvements on the ratio of gate operation time to decoherence times may be achieved using adiabatic passage schemes to eliminate or minimize the virtual amplitude generated in the cavity mode and the valence-band hole states. The Hamiltonian of Eq. (4) can also be used to achieve quantum state transfer from one QD to another, along the lines described in the context of atomic cavity QED.

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[15] We point out that strong coupling of a QD to a single cavity mode, which is a principal feature of the proposed scheme, is still not observed experimentally. However, recent experiments in QDs coupled to microdisks [12] and silica microspheres [16] indicate that strong coupling could be observed in the near future.
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